Chapter 18 – Logarithms to Base e and Power of e (LL scale)

18.1 The form of the LL scale.

The LL scales are found on the body of the Slide Rule and are used in conjunction with the D (or C scale). The LL_3, LL_2, and LL_1 (some also have an LL_0) scales are in black and give the positve powers of e. A number of slide rules have the inverted LL_{03}, LL_{02}, and LL_{03} (some also have LL_{00}) scales in red (reading from right to left) to give the negative powers of e. Note that the decimal point is given with all numbers marked on these scales. In the following, we have dealt with Slide Rules with 3, 6, 8 LL scales.

Example: $e^{1.52} = 4.75$

1. Set the hair line over 1.52 on the D (or C scale).
2. Under the hair line read off 4.57 on the LL_3 (i.e. e^x) scale as the answer.

Note:

(a) we read $e^{0.152} = 1.164$ on the LL_2 scale (i.e. $e^{0.1x}$) scale.
$e^{0.0152} = 1.0153$ on the LL_1 scale (i.e. $e^{0.01x}$) scale.
and $e^{0.00152} = 1.00152$ on the LL_0 scale (i.e. $e^{0.001x}$) scale.

For a on the D scale between	***e^a read off***
1 and 10	LL_3 scale
0.1 and 1	LL_2 scale
0.01 and 0.1	LL_1 scale
0.001 and 0.01	LL_0 scale

(b) As $e^N = x$ is equivalent to $\log_e x = N$ we can either say (for example) evaluate $e^{2.5}$, or solve $\log_e x = 2.5$ for x.

Exercise 18(a)

(i)	$e^{4.3} =$	(v)	$e^{0.062} =$
(ii)	$e^{7} =$	(vi)	$e^{0.48} =$
(iii)	$e^{0.95} =$	(vii)	$e^{0.003} =$
(iv)	$e^{0.035} =$	(viii)	$e^{0.0051} =$

Solve the following for x:

(ix)	$\text{Log}_e x = 3.1$	(xi)	$\text{Log}_e x = 0.007$
(x)	$\text{Log}_e x = 0.62$	(xii)	$\text{Log}_e x = 0.049$

18.3 Negative Powers of e

Example 1: $e^{-7.9} = 0.00037$

1. Set the hair line over 7.9 on the D (or C) scale.
2. Under the hair line read off 0.00037 on the LL_{03} (i.e. e^{-x}) scale.

Note:

(a) we read $e^{-0.79} = 0.454$ on the LL_{02} scale (i.e. $e^{-0.1x}$) scale.
$e^{-0.079} = 0.924$ on the LL_{01} scale (i.e. $e^{-0.01x}$) scale.
and $e^{-0.0079} = 0.99213$ on the LL_{00} scale (i.e. $e^{-0.001x}$) scale.

For a on the D scale between	***e^a read off***
1 and 10	LL_{03} scale
0.1 and 1	LL_{02} scale
0.01 and 0.1	LL_{01} scale
0.001 and 0.01	LL_{00} scale

(b) For Slide Rules without the e^{-x} scales we can still obtain negative powers of e by using the fact $e^{-x} = \dfrac{1}{e^x}$.

Example 2: $e^{-0.79} = .454$

1. Set the hair line over 0.79 on the D (or C) scale).
2. Under the hair line read off 2.205 on the LL_2 scale as the answer of $e^{0.79}$.
3. Reset the hair line over 2.205 on the C scale.
4. Under the hair line read off 0.454 on the CI scale as the value for $\dfrac{1}{e^{0.79}}$ (i.e. $e^{-0.79}$)

Exercise 18(b)

(i) $e^{-2.1} =$ (iv) $e^{-0.19} =$ (vii) $e^{-0.0019} =$

(ii) $e^{-9} =$ (v) $e^{-0.062} =$ (viii) $e^{-0.0083}$

(iii) $e^{-0.4} =$ (vi) $e^{-0.024} =$

Solve the following for x:

(ix) $\text{Log}_e x = -2$ (xi) $\text{Log}_e x = -0.068$

(x) $\text{Log}_e x = -0.4$ (xii) $\text{Log}_e x = -0.0032$

18.4 Miscellaneous Powers of e

A. Very Small Powers of e can be approximated as follows:

for any x $e^x \approx 1 + x$

This can be easily verified and is useful when a Slide Rule does not have LL_0 and LL_{00} scales.

e.g. $e^{0.0027} \approx 1 + 0.0027 = 1.0027$,

$e^{-0.0018} \approx 1 - 0.0018 = 0.9982$, etc.

B. Very Large Powers of e (i.e. e^x for x > 10).

Example 1: $e^{15} = 3{,}320{,}000$

Express $e^{15} = (e^5)^3$

$= (149)^3$ (obtain $e^5 = 149$ using D and LL_3 scales)

= 3.320,000 (cube using D and K scales)

OR

Express $e^{15} = e^{10+5} = e^{10}\, e^5$

= 22,000 x 149

= 3,380,000

(Note, discrepancies may occur in the third or forth significant figures from the different methods due to limitations of accuracy obtainable with various scales.)

Example 2: $e^{-15} = 3.01 \times 10^{-7}$

Express $e^{-15} = \left(e^{-5}\right)^3$

$= (0.0067)^3$

$= (6.7 \times 10^{-3})^3$

$= 301 \times 10^{-9} = 3.01 \times 10^{-7}$

C. Some other powers of e which may be worthy of note are covered in the following table.

Set the H.L. over		Under H.L. on LL scales read	Under H.L on LL_0 scales read
a on	A scale	$e^{\sqrt{a}}$	$e^{-\sqrt{a}}$
a	K	$e^{\sqrt[3]{a}}$	$e^{-\sqrt[3]{a}}$
θ	S	$e^{\sin\theta}$	$e^{-\sin\theta}$
θ (red)	S	$e^{\cos\theta}$	$e^{-\cos\theta}$
θ	T_1 or T_2	$e^{\tan\theta}$	$e^{-\tan\theta}$

a	DI(CI)	$\sqrt[a]{e}$	$\frac{1}{\sqrt[a]{e}}$
a	BI	$e^{\frac{1}{\sqrt{a}}}$	$e^{-\frac{1}{\sqrt{a}}}$
a	DF	$e^{\frac{a}{\pi}}$	$e^{-\frac{a}{\pi}}$
a	P	$e^{\sqrt{1-a^2}}$	$e^{-\sqrt{1-a^2}}$
a	W	e^{a^2}	e^{-a^2}

18.5 Logarithms to Base e

To find $\log_e N$ (or ln N as we often write it), the hair line is set over N on the LL scale and under the hair line on the D (or C) scale ln N is found.

The sign and position of the decimal point is dependent upon which LL scale N is located on.

Example 1: ln 4.57 = 1.52

(i.e. the steps shown in Fig. 18-1 in reverse order)

1. Set the hair line over 4.57 on the LL_3 Scale.
2. Under the hair line read off 1.52 on the D scale as the answer.

Example 2: ln 1.02 = 0.0198

1. Set the hair line over 1.02 on the LL_1 scale.
2. Under the hair line read off 0.0147 on the D scale as the answer.
   (It reads as 0.0147 on the D scale, as 1.02 is found on the LL_1 scale (i.e. $e^{0.01x}$)).

Example 3: ln 0.6 = -0.511

1. Set the hair line over 0.6 on the LL_{02} scale.
2. Under the hair line read off –0.511 on the D scale as the answer.
   (It reads as –0.511 on the D scale, as 0.6 is found on the LL_{02} scale (i.e. $e^{-0.1x}$)).

Note:

(a)

Found on the –	ln is read off the D scale as a value between –		
LL_3	1.0	and	10.0
LL_2	0.1	and	1.0
LL_1	0.01	and	0.1
LL_0	0.001	and	0.01
LL_{00}	-0.001	and	-0.01
LL_{01}	-0.01	and	-0.1
LL_{02}	-0.1	and	1.0
LL_{03}	-1.0	and	-10.0

(We do not attempt to write the logarithm of numbers less than 1 with a negative characteristic and positive mantissa, but leave it as a negative number.)

(b) For Slide Rules without e^{-x} scales we can still obtain natural logarithms of numbers less than 1, by using the fact $\ln x = -\ln \frac{1}{x}$.

Example: ln 0.6 = -0.511

(i.e. $\ln 0.6 = -\ln\frac{1}{0.6}$)

1. Set the hair line over 0.6 on the C scale.
2. Under the hair line read off 1.677 on the CI scale as the value for $\frac{1}{0.6}$.
3. Reset the hair line over 1.667 on the LL_2 scale.
4. Under the hair line read off –0.511 on the D scale as the value for ln 0.6.

(c) As ex = N is equivalent to ln N = x, we can either say, (for example) solve ex = 2.5 for x or evaluate ln 2.5.

(d) To solve $\sqrt[x]{e} = N$ for x, we note it can be written as $e^{\frac{1}{x.}} = N.$

i.e. $\ln N = \frac{1}{x}$

Thus to find what root of e is equal to a number N, we set the hair line over N on the LL scales and read off x on the DI scale.

(e) Logarithms of numbers near 1 are very small positive or negative values, depending upon whether the number is greater or less than 1. (ln 1 = 0). The logarithms of such numbers can be approximated, as follows:
ln 1.0023 $\approx$ 1.0023 – 1 = 0.0023
ln 0.9984 $\approx$ 0.9984 – 1 = -0.0016 (etc.)
This is useful for Slide Rules with out LL_2 and LL_{03} scales.

Exercise 18(d)

(i)	ln 9.5	(vii)	ln 1.005	(xiii)	ln 0.63
(ii)	ln 170	(viii)	ln 1.0019	(xiv)	ln 0.422
(iii)	ln 2.4	(ix)	ln 0.9985	(xv)	ln 0.035
(iv)	ln 1.14	(x)	ln 0.992	(xvi)	ln 0.0002
(v)	ln 1.07	(xi)	ln 0.95		
(vi)	ln 1.01	(xii)	ln 0.98		

Solve the following for x:

(xvii) $e^x = 4{,}000$
(xviii) $e^x = 1.06$
(xix) $e^x = 0.1$
(xx) $e^x = 0.9$

(xxi) $\sqrt{e} = 1.65$
(xxii) $\sqrt{e} = 1.0202$